Formalizations and Implementations of Monotonicity Reasoning

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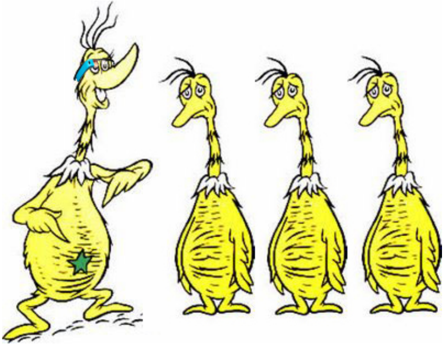
This is an entry for a United States National Science Foundation contest on mathematics outreach for the general public.

Other examples of arrows



Some "givens"

animal
Sneetch
Star-Belly Sneetch

move
dance
waltz

Other examples of arrows

Some "givens"

animal
Sneetch
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move
dance
waltz

Let's talk about a situation where

all Sneetches dance.

Which one would be true?

- all Star-Belly Sneetches dance
- all animals dance

Other examples of arrows

Some "givens"

animal
Sneetch
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move
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waltz

- all Star-Belly Sneetches dance   true
- all animals dance   false

We write

all Sneetches$\downarrow$ dance

Some "givens"

animal
Sneetch
Star-Belly Sneetch

move
dance
waltz

all Sneetches$\downarrow$ dance

What arrow goes on "dance"?

- all Sneetches waltz
- all Sneetches move

Other examples of arrows

Some "givens"

animal
Sneetch
Star-Belly Sneetch

move
dance
waltz

We write

all Sneetches$\downarrow$ dance$\uparrow$

Some "givens"

animal
Sneetch
Star-Belly Sneetch

move
dance
waltz

Let's put the arrows on the words Sneetches and dance.

1. No Sneetches dance.
2. Most Sneetches dance.
3. If you play loud enough music, any Sneetch will dance.
4. Any Sneetch in Zargonia would prefer to live in Yabistan.
5. If any Sneetch dances, McBean will dance, too.

Other examples of arrows

Some "givens"

animal
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Let's put the arrows on the words Sneetches and dance.

1. No Sneetches$\downarrow$ dance$\downarrow$.
2. Most Sneetches$=$ dance$\uparrow$. (no arrow)
3. If you play loud enough music, any Sneetch will dance.
4. Any Sneetch in Zargonia would prefer to live in Yabistan.
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Some "givens"

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Sneetch	dance
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5. If any Sneetch$\downarrow$ dances$\downarrow$, McBean will dance$\uparrow$, too.

Arrows found by MonaLog: see Hai Hu's talk tomorrow

No$^{\uparrow}$ man$^{\downarrow}$ walks$^{\downarrow}$

Every$^{\uparrow}$ man$^{\downarrow}$ and$^{\uparrow}$ some$^{\uparrow}$ woman$^{\uparrow}$ sleeps$^{\uparrow}$

Every$^{\uparrow}$ man$^{\downarrow}$ and$^{\uparrow}$ no$^{\uparrow}$ woman$^{\downarrow}$ sleeps$^{=}$

If$^{\uparrow}$ some$^{\downarrow}$ man$^{\downarrow}$ walks$^{\downarrow}$, then$^{\uparrow}$ no$^{\uparrow}$ woman$^{\downarrow}$ runs$^{\downarrow}$

Every$^{\uparrow}$ man$^{\downarrow}$ does$^{\downarrow}$ n't$^{\uparrow}$ hit$^{\downarrow}$ every$^{\downarrow}$ dog$^{\uparrow}$

No$^{\uparrow}$ man$^{\downarrow}$ that$^{\downarrow}$ likes$^{\downarrow}$ every$^{\downarrow}$ dog$^{\uparrow}$ sleeps$^{\downarrow}$

Most$^{\uparrow}$ men$^{=}$ that$^{=}$ every$^{=}$ woman$^{=}$ hits$^{=}$ cried$^{\uparrow}$

Every$^{\uparrow}$ young$^{\downarrow}$ man$^{\downarrow}$ that$^{\uparrow}$ no$^{\uparrow}$ young$^{\downarrow}$ woman$^{\downarrow}$ hits$^{\uparrow}$ cried$^{\uparrow}$

The arrows are sites for inference
At this point, we might wonder:

- What would logic look like if simple inferences like the ones we've seen played a central role, not a supporting part.
- How much of human reasoning is monotonicity reasoning anyways?

Fix a set of nouns.

Sentences

$$\text{all } p \ q$$

for $p, q \in nouns$.

Semantics: models

A model $\mathcal{M}$ is a set M together with an interpretation function

$$[\![\ ]\!] : nouns \to \mathcal{P}(M).$$

(Here $\mathcal{P}(M)$ is the set of subsets of M.)
Then we say that

$$\mathcal{M} \models \text{all } p \ q \quad \text{iff} \quad [\![p]\!] \subseteq [\![q]\!].$$

We use ϕ for sentences and Γ for sets thereof.

We say that

$$\mathcal{M} \models \Gamma$$

if $\mathcal{M} \models \phi$ for each $\phi \in \Gamma$.

We then say that

$$\Gamma \models \phi$$

if for all models $\mathcal{M}$,

$$\mathcal{M} \models \Gamma \text{ implies } \mathcal{M} \models \phi.$$

We have a logical system.

It uses the following two rules:

$$\frac{}{\text{all } x\ x}\ \textsc{axiom} \qquad\qquad \frac{\text{all } x\ y \quad \text{all } y\ z}{\text{all } x\ z}\ \textsc{barbara}$$

Recasting (BARBARA)

$$\text{all } x^{\downarrow}\ y^{\uparrow}$$

We have a logical system.

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Recasting (Barbara)

$$\text{all } x^{\downarrow}\ y^{\uparrow}$$

Theorem (Soundness/Completeness/Complexity)

$\Gamma \models \phi$ *iff* $\Gamma \vdash \phi$.

Moreover, this relation is decidable in polynomial time, and if $\Gamma \nvdash \phi$, *then there are "small counter-models".*

We can extend a number of orthogonal directions:

- by adding names.
- by adding noun complements $\overline{p}$, $\overline{q}$, ...,
  and then using these in sentences.
- by adding verbs (e.g., see), and then forming terms
  - see all p
  - see some p
    Then these terms are a productive construct:
    e.g., see all (like all cats)

  and then the terms can be used in the basic sentences,
  as in all (see some dog) (chase all cats)

A simple fragment

$\textsc{t}$ is for "terms"; their denotations are sets

$$\textsc{S} \to \textsc{Det}\ \textsc{T}\ \textsc{T}$$

$$\textsc{T} \to \textsc{V}\ \textsc{Det}\ \textsc{T}$$

$$\textsc{T} \to \text{dogs}, \text{cats}, \ldots$$

$$\textsc{Det} \to \text{all}, \text{some}, \text{no}$$

$$\textsc{V} \to \text{see}, \text{chase}, \ldots$$

$$
\begin{aligned}
\mathrm{S} &\rightarrow \mathrm{DET}\ \mathrm{T}\ \mathrm{T} \\
\mathrm{T} &\rightarrow \mathrm{V}\ \mathrm{DET}\ \mathrm{T} \\
\mathrm{T} &\rightarrow \textsf{dogs}, \textsf{cats}, \ldots \\
\mathrm{DET} &\rightarrow \textsf{all}, \textsf{some}, \textsf{no} \\
\mathrm{V} &\rightarrow \textsf{see}, \textsf{chase}, \ldots
\end{aligned}
$$

Examples

all skunks mammals

no skunks mammals

all skunks see all mammals

all skunks see all fear all mammals

Examples

all$\uparrow$ skunks$\downarrow$ mammals$\uparrow$

no$\uparrow$ skunks$\downarrow$ mammals$\downarrow$

all$\uparrow$ skunks$\downarrow$ see$\uparrow$ some$\uparrow$ mammals$\uparrow$

all$\uparrow$ skunks$\downarrow$ see$\uparrow$ all$\uparrow$ fear$\downarrow$ all$\downarrow$ mammals$\uparrow$

$$\textsc{s}^{\uparrow} \to \textsf{all}^{\uparrow}\ \textsc{t}^{\downarrow}\ \textsc{t}^{\uparrow} \mid \textsf{some}^{\uparrow}\ \textsc{t}^{\uparrow}\ \textsc{t}^{\uparrow} \mid \textsf{no}^{\uparrow}\ \textsc{t}^{\downarrow}\ \textsc{t}^{\downarrow}$$

$$\textsc{t}^{\uparrow} \to \textsc{v}^{\uparrow}\ \textsf{all}^{\uparrow}\ \textsc{t}^{\downarrow} \mid \textsc{v}^{\uparrow}\ \textsf{some}^{\uparrow}\ \textsc{t}^{\uparrow} \mid \textsc{v}^{\downarrow}\ \textsf{no}^{\uparrow}\ \textsc{t}^{\downarrow} \mid \textsf{dogs}^{\uparrow} \mid \textsf{cats}^{\uparrow} \mid \cdots$$
$$\textsc{det}^{\uparrow} \to \textsf{all}^{\uparrow} \mid \textsf{some}^{\uparrow} \mid \textsf{no}^{\uparrow}$$
$$\textsc{v}^{\uparrow} \to \textsf{see}^{\uparrow} \mid \textsf{chase}^{\uparrow} \mid \cdots$$

$$\textsc{t}^{\downarrow} \to \textsc{v}^{\downarrow}\ \textsf{all}^{\downarrow}\ \textsc{t}^{\uparrow} \mid \textsc{v}^{\downarrow}\ \textsf{some}^{\downarrow}\ \textsc{t}^{\downarrow} \mid \textsc{v}^{\uparrow}\ \textsf{no}^{\downarrow}\ \textsc{t}^{\uparrow} \mid \textsf{dogs}^{\downarrow} \mid \textsf{cats}^{\downarrow} \mid \cdots$$
$$\textsc{det}^{\downarrow} \to \textsf{all}^{\downarrow} \mid \textsf{some}^{\downarrow} \mid \textsf{no}^{\downarrow}$$
$$\textsc{v}^{\downarrow} \to \textsf{see}^{\downarrow} \mid \textsf{chase}^{\downarrow} \mid \cdots$$

Clearly it's a bit excessive to list similar things twice.
So we use d as a variable ranging over Pol $= \{\uparrow, \downarrow\}$.

We also introduce an operation $-$: Pol $\to$ Pol in the obvious way.

And then our rules simplify to:

$$\begin{array}{l}
\mathrm{S}^{d} \to \mathsf{all}^{d}\ \mathrm{T}^{-d}\ \mathrm{T}^{d} \mid \mathsf{some}^{d}\ \mathrm{T}^{d}\ \mathrm{T}^{d} \mid \mathsf{no}^{d}\ \mathrm{T}^{-d}\ \mathrm{T}^{-d} \\
\mathrm{T}^{d} \to \mathrm{V}^{d}\ \mathsf{all}^{d}\ \mathrm{T}^{-d} \mid \mathrm{V}^{d}\ \mathsf{some}^{d}\ \mathrm{T}^{d} \mid \mathrm{V}^{-d}\ \mathsf{no}^{d}\ \mathrm{T}^{-d} \mid \mathsf{dogs}^{d} \mid \mathsf{cats}^{d} \mid \cdots \\
\mathrm{DET}^{d} \to \mathsf{all}^{d} \mid \mathsf{some}^{d} \mid \mathsf{no}^{d} \\
\mathrm{V}^{d} \to \mathsf{see}^{d} \mid \mathsf{chase}^{d} \mid \cdots
\end{array}$$

Most Sneetches$^{=}$ dance$^{\uparrow}$

When we add most, we see the need for a third polarity $=$.

And then we would extend the $-$ operation by taking $-=$ to be $=$.

The extra productions which we want are

$$\begin{aligned} \mathrm{S}^{d} &\to \mathsf{most}^{d}\ \mathrm{T}^{=}\ \mathrm{T}^{d} \\ \mathrm{T}^{d} &\to \mathrm{V}^{d}\ \mathsf{most}^{d}\ \mathrm{T}^{=} \\ \mathrm{DET}^{d} &\to \mathsf{most}^{d} \end{aligned}$$

(1) some$^{\uparrow}$ dog$^{\uparrow}$ hit$^{\uparrow}$ some$^{\uparrow}$ cat$^{\uparrow}$

(2) some$^{\uparrow}$ dog$^{\uparrow}$ kissed$^{\downarrow}$ no$^{\uparrow}$ cat$^{\downarrow}$

(3) most$^{\uparrow}$ dog$^{=}$ hit$^{\downarrow}$ no$^{\uparrow}$ cat$^{\downarrow}$

(4) no$^{\uparrow}$ dog$^{\downarrow}$ hit$^{\uparrow}$ no$^{\downarrow}$ cat$^{\uparrow}$

(5) at most two$^{\uparrow}$ dog$^{\downarrow}$ chased$^{\uparrow}$ at most three$^{\downarrow}$ cats$^{\uparrow}$

knowledge base for nouns, transitive verbs, determiners, and numbers

dog $\leq$ animal	kiss $\leq$ touch	every $\leq$ most
cat $\leq$ animal	hit $\leq$ touch	most $\leq$ some
poodle $\leq$ dog	thrash $\leq$ hit	one $\leq$ two
siamese $\leq$ cat	hit vigorously $\leq$ hit	two $\leq$ three
bird $\leq$ scooter	hit lightly $\leq$ hit	three $\leq$ four

What do you think?

Is this valid?

$$\frac{\textit{All skunks are mammals}}{\textit{All who fear all who respect all skunks fear all who respect all mammals}}$$

It follows, using an interesting antitonicity principle:

$$\frac{\textit{All skunks are mammals}}{\textit{All who respect all mammals respect all skunks}}$$

What do you think?

It follows, using an interesting antitonicity principle:

$$\dfrac{\dfrac{\textit{All skunks are mammals}}{\textit{All who respect all mammals respect all skunks}}}{\textit{All who fear all who respect all skunks fear all who respect all mammals}}$$

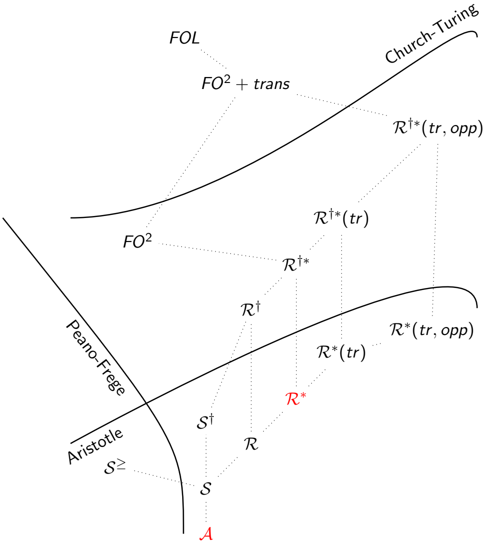
Indeed, in this logic it would be done in one step from the axiom

all (fear all (respect all skunks)) (fear all (respect all skunks$^{\uparrow}$))

and the background information

$$\text{skunks} \;\leq\; \text{mammals}$$

A Map of Natural Logics



first-order logic

FO^2 + "R is trans"

2 variable FO logic

$\dagger$ adds full N-negation

$\mathcal{R}^*(tr)$ + opposites

$\mathcal{R}^*$ + (transitive) comparative adjs

$\mathcal{R}$ + relative clauses

$\mathcal{S}$ + full N-negation

$\mathcal{R}$ = relational syllogistic

$\mathcal{S}^{\geq}$ adds $|p| \geq |q|$

$\mathcal{S}$: all/some/no p are q

$\mathcal{A}$: all p are q

But all of this is not so useful for real-world NLI

One of the many problems

Text doesn't come to us according to a grammar.

So to handle this in a way to have logic and semantics
but not a simple form of grammar,
we turn to categorial grammar.

Indeed to use off-the-shelf parsers, we use CCG.

The last part of this talk is a very high-level description of
MonaLog,
and you'll hear more in Hai Hu's talk tomorrow.

To derive the $\uparrow$ and $\downarrow$ polarities, we need to change the entire architecture of CG,
and indeed to change everything about the semantics,
going from sets to preorders.

(For preorders and their theory, see the end of this slide set.)

For example, standard CG has function types $X \to Y$.

In the preorder enrichment, we have

- $X \xrightarrow{+} Y$ (monotone functions)
- $X \xrightarrow{-} Y$ (antitone functions)
- $X \xrightarrow{\cdot} Y$ (all functions)

Preorder enrichment of grammar

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We start with

$$
\begin{array}{lcl}
\mathbb{P}_e & = & \text{the flat order on some set} \\
\mathbb{P}_t & = & 2 \\
\mathbb{P}_{num} & = & \mathbb{N}
\end{array}
$$

A lexicon

By flatness, $e \to t$ is the same as $e \xrightarrow{+} t$ and $e \xrightarrow{-} t$

item	category	type
Fido, Felix	NP	e
cat, dog	N	$pr = (e \to t)$
swim, run	IV = S\NP	pr
chase, see, hit, kiss	TV = IV/NP	$e \to pr$
every	DET = NP/N	$pr \xrightarrow{-} np^{+}$
some	NP/N	$pr \xrightarrow{+} np^{+}$
no	NP/N	$pr \xrightarrow{-} np^{-}$
most	NP/N	$pr \xrightarrow{\cdot} np^{+}$
didn't	IV/IV	$pr \xrightarrow{-} pr$
	TV/TV	$(e \to pr) \xrightarrow{-} (e \to pr)$
and	X/(X\X)	$x \xrightarrow{+} (x \xrightarrow{+} x)$
one, two, three	NUM	num
more than	DET/NUM	$num \xrightarrow{-} (pr \xrightarrow{+} np^{+})$
less than	DET/NUM	$num \xrightarrow{+} (pr \xrightarrow{-} np^{-})$
if ... then ...	(S\S)/S	$t \xrightarrow{-} (t \xrightarrow{+} t)$

Polarizing the CCG rules

Unpolarized rules

$$\frac{u : x \to y \quad v : x}{uv : y}\; > \qquad \frac{u : x}{\mathrm{T}u : (x \to y) \to y}\; \mathrm{T} \qquad \frac{u : x \to y \quad v : y \to z}{\mathrm{B}uv : (x \to z)}\; \mathrm{B}$$

Polarized rules

$$\frac{u^{d} : x \xrightarrow{m} y \quad v^{md} : x}{(uv)^{d} : y}\; > \qquad \frac{u^{md} : x}{(\mathrm{T}u)^{d} : (x \xrightarrow{m} y) \xrightarrow{+} y}\; \mathrm{T} \qquad \frac{u^{md} : x \xrightarrow{n} y \quad v^{md} : y \xrightarrow{n} z}{(\mathrm{B}uv)^{d} : (x \xrightarrow{mn} z)}\; \mathrm{B}$$

$$\frac{u^{md} : e \to b}{(r_m u)^{d} : np^{m} \xrightarrow{+} b}\; \mathrm{K} \qquad \frac{u^{d} : x \xrightarrow{m} y}{u^{d} : x \xrightarrow{\cdot} y}\; \mathrm{M} \qquad \frac{u^{=} : x \xrightarrow{m} y}{u^{d} : x \xrightarrow{m} y}\; \mathrm{W}$$

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The $>$ in the application rule is function application.

The T in the type-raising rule is the Montague lift.

The B in the type-raising rule is function composition, backwards.

The r_m in the K rule is from our refinement of the Justification Theorem.

In the M rule, we have a trivial inclusion.

The W rule is trivial.

Polarizing the CCG rules

Unpolarized rules

$$\frac{u : x \to y \quad v : x}{uv : y}\ > \qquad \frac{u : x}{\mathrm{T}u : (x \to y) \to y}\ \mathrm{T} \qquad \frac{u : x \to y \quad v : y \to z}{\mathrm{B}uv : (x \to z)}\ \mathrm{B}$$

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We combine markings and polarities as in the table below:

md	$+$	$-$	$\cdot$
$\uparrow$	$\uparrow$	$\downarrow$	$=$
$\downarrow$	$\downarrow$	$\uparrow$	$=$
$=$	$=$	$=$	$=$

$$\dfrac{\text{Fido}^{\uparrow} : e \qquad \dfrac{\text{didn't}^{\uparrow} : pr \stackrel{-}{\to} pr \qquad \dfrac{\dfrac{\text{chase}^{\downarrow} : e \stackrel{+}{\to} pr}{\text{chase}^{\downarrow} : np^{+} \stackrel{+}{\to} pr}\,\text{K} \qquad \dfrac{\text{every}^{\downarrow} : pr \stackrel{-}{\to} np^{+} \quad \text{cat}^{\uparrow} : pr}{\text{every cat}^{\downarrow} : np^{+}}\,>}{\text{chase every cat}^{\downarrow} : pr}\,>}{\text{didn't chase every cat}^{\uparrow} : pr}\,>}{\text{Fido didn't chase every cat}^{\uparrow} : t}\,<$$

$$\dfrac{\dfrac{\text{some}^{\uparrow} : pr \stackrel{+}{\to} np^{+} \quad \text{dog}^{\uparrow} : pr}{\text{some dog}^{\uparrow} : pr \stackrel{+}{\to} t}\,> \qquad \dfrac{\dfrac{\text{chased}^{\downarrow} : e \stackrel{+}{\to} pr}{\text{chased}^{\uparrow} : np^{-} \stackrel{+}{\to} pr}\,\text{K} \qquad \dfrac{\text{no}^{\uparrow} : pr \stackrel{-}{\to} np^{-} \quad \text{cat}^{\downarrow} : pr}{\text{no cat}^{\uparrow} : np^{-}}\,>}{\text{chased no cat}^{\uparrow} : pr}\,>}{\text{some dog chased no cat}^{\uparrow} : t}\,>$$

$$
\dfrac{\mathrm{F}^{\uparrow} : e \qquad \dfrac{\dfrac{\mathrm{ch}^{\uparrow} : e \xrightarrow{+} pr}{\mathrm{ch}^{\uparrow} : np^{+} \xrightarrow{+} pr}\,\mathrm{K} \qquad \dfrac{\text{some cat}^{\uparrow} : np^{+} \qquad \dfrac{\text{and} : np^{+} \xrightarrow{+} (np^{+} \xrightarrow{+} np^{+}) \quad \text{some arm}^{\uparrow} : np^{+}}{\text{and some arm} : np \xrightarrow{+} np}\,>}{\text{some cat and some armadillo}^{\uparrow} : np}\,<}{\text{chased some cat and some armadillo}^{\uparrow} : pr}\,>}{\text{Fido chased some cat and some armadillo}^{\uparrow} : t}\,<
$$

Definition

A preorder is a pair $\mathbb{P} = (P, \leq)$ consisting of a set P together with a relation $\leq$ which is
reflexive and transitive.

This means that the following hold:

- $p \leq p$ for all $p \in P$.
- If $p \leq q$ and $q \leq r$, then $p \leq r$.

The set of truth values $\mathbb{2} = \{\mathsf{T}, \mathsf{F}\}$ is a preorder, with $\mathsf{F} \leq \mathsf{T}$.

The set of real numbers $\mathbb{R}$ is a preorder, with the usual $\leq$.

Definition

For any preorder $\mathbb{P}$ and any set X,
we have a new preorder called $X \to \mathbb{P}$.

The domain of this preorder is the function set

$$X \to P$$

The order on $\mathbb{P}^X$ is the pointwise order:

$$f \leq g \text{ iff for all } x \in X,\ fx \leq_{\mathbb{P}} gx.$$

Definition

For any preorder $\mathbb{P}$, there is an opposite preorder $\mathbb{P}^{op}$.
Its domain set is P, the same domain set as for $\mathbb{P}$.

$$p \leq q \text{ in } \mathbb{P}^{op} \quad \text{iff} \quad q \leq p \text{ in } \mathbb{P}$$

Definition

For any preorder $\mathbb{P}$, there is an flattened version $\mathbb{P}^{\flat}$.
Its domain set is P, the same domain set as for $\mathbb{P}$.

$$p \leq q \text{ in } \mathbb{P}^{\flat} \quad \text{iff} \quad p = q$$

Definition

For any preorders $\mathbb{P}$ and $\mathbb{Q}$, there is a product preorder $\mathbb{P} \times \mathbb{Q}$.
Its domain set is the cartesian product $P \times Q$.

$$(p, q) \leq (p', q') \text{ in } \mathbb{P} \times \mathbb{Q} \quad \text{iff} \quad p \leq p' \text{ in } \mathbb{P}, \text{ and } q \leq q' \text{ in } \mathbb{Q}$$

An algebraic expression like

$$z - (v + |w|)$$

is increasing in z, and decreasing in v, and
there's nothing we can say about w.

If we assume

- $z_1 \leq z_2$
- $v_2 \leq v_1$
- $w_2 = w_1$

Then we are entitled to conclude

$$z_1 - (v_1 + w_1) \leq z_2 - (v_2 + w_2)$$

We had

$$z - (v + |w|)$$

We would write

$$\frac{v^{\downarrow} \quad w^{=} \quad z^{\uparrow}}{(z - (v + |w|))^{\uparrow}} \tag{1}$$

The responsible parties here are the facts that

$+ : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$	is increasing (monotone) in both arguments
$- : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$	is increasing in the first argument
	and decreasing (antitone) in the second argument
$\lvert\ \rvert : \mathbb{R} \to \mathbb{R}$	is neither

And we can write (1) as
$v, w, z \mapsto z - (v + w)$ is an increasing function

$$\mathbb{R}^{op} \times \mathbb{R}^{\flat} \times \mathbb{R} \to \mathbb{R}$$

Historical influences on this project

- van Benthem 1986, 1991: combine vanilla CG with inference
- Nairn, Condoravdi, and Karttunen 2006:
  something similar (!),
  but not noticed as such,
  not using CG, and not aimed at the same issues
- Steedman: CCG, a working system
- Dowty 1994: internalization of inferential features
  in the type system
- MacCartney and Manning 2009: get something to work.

Logic-based approaches to NLI

- Tableau (Muskens, Abzianidze)
- Translation to a richer logical form,
  then call a theorem prover
  (Bekki, Martinez-Gomez, Mineshima, Yanaka)
- Using a Graphical Knowledge Representation Parser
  (Crouch, de Paiva, Kalouli)
- Natural Logic: monotonicity calculus + special rules
  (Icard, Pratt-Hartmann, M., Hu)

My wonderful collaborators on these topics

Jörg Endrullis
Jason Hemann
Hai Hu
Thomas Icard
Katerina Kalouli
Alex Kruckman
Tri Lai
Valeria de Paiva
Ian Pratt-Hartmann
Charlotte Raty
Livy Real
Cameron Swords
Selcuk Topal
Will Tune
Chloe Urbanski
Michael Wollowski

I hope you find our workshop to be stimulating!